CBCS SCHEME

USN | | | | | | | | | | |

15MAT31

Third Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the Fourier series expansion of $f(x) = 2x - x^2$ in (0, 3). **(08 Marks)**

b. The turning moment T is given for a series of values of the Crank angle $\theta^o = 75^o$

θ^o	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Obtain the first four terms in a series of sires to represent T. Also calculate T for $\theta = 75^o$. **(08 Marks)**

OR

2 a. Obtain Fourier series for the function f(x) given by

$$f(x) = \begin{cases} \pi + x & -\pi < x < 0 \\ \pi - x & 0 \le x < \pi \end{cases}. \quad \text{Hence deduce } \frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}.$$ **(08 Marks)**

b. i) Define Half range Fourier sine series of f(x) **(02 Marks)**

ii) Find the half range Cosine series of $f(x) = x^2$ in the range $0 \le x \le \pi$. **(06 Marks)**

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for} |x| < 1 \\ 0 & \text{for} |x| > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \left(\frac{\sin x}{x}\right) dx$. **(06 Marks)**

b. Find the inverse sine transform of $F_s(\alpha) = \begin{cases} 1 & 0 \le \alpha < 1 \\ 2 & 1 \le \alpha < 2 \\ 0 & \alpha \ge 2 \end{cases}$ **(05 Marks)**

c. Find the inverse Z- transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(05 Marks)**

OR

4 a. Find the Fourier Sine transform of $f(x) = e^{-|x|}$ and hence show that

$$\int_0^{\infty} \frac{x \sin mx}{1 + x^2} dx = \frac{\pi}{2} e^{-m}, m > 0.$$ **(06 Marks)**

b. Find the Z-transform of i) $\cosh n\theta$ ii) $\sinh n\theta$. **(05 Marks)**

c. Using the Z- transform, solve $u_{n+2} + u_n = 0$ given $u_0 = 1$, $u_1 = 2$. **(05 Marks)**

Module-3

5 a. Find the correlation coefficient between x and y

x	2	4	6	8	10
y	5	7	9	8	11

(06 Marks)

b. Fit the curve of the form $y = a + bx + cx^2$ to the following data :

x	0	1	2	3	4
y	-4	-1	4	11	20

(05 Marks)

c. Find the root of the equation $2x - \log_e x = 7$ using Regula-Falsi method. Carry out 3 iteration. **(05 Marks)**

OR

6 a. If θ is the angle between the two regression lines, show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\left[\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right]$.
Explain the significance when r = 0 and r = ±1. **(06 Marks)**

b. Use the method of least squares fit a curve of the form $y = a\,e^{bx}$ for the following data :

x	0	2	4	6	8
y	150	63	28	12	5.6

(05 Marks)

c. Find the real root of the equation $x^4 - x = 10$ by using Newton –Raphson method, carryout 3 iteration. **(05 Marks)**

Module-4

7 a. Find f(x), using Newton's interpolation formula

x	0	1	2	3	4
f(x)	-5	-10	-9	4	35

(06 Marks)

b. Find f(g) : Using Newton's divided difference formula

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(05 Marks)

c. Evaluate, using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule for $\int_0^{\pi/2}\sqrt{\sin x}\,dx$ by taking 6 intervals. **(05 Marks)**

OR

8 a. A curve passing through the points (0, 18) (1, 10) (3, -18) and (6, 90). Find f(x), using Lagrange's interpolation formula. **(05 Marks)**

b. Evaluate, using Weddle's rule $\int_0^6 \frac{e^x}{1+x}dx$ by taking 7 ordinates. **(05 Marks)**

c. The area 'A' of a circle of diameter 'd' is given for the following values

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105. **(06 Marks)**

Module-5

9 a. By using Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x\, dy]$ where C is the plane triangle enclosed by the lines $y = 0$; $x = \frac{\pi}{2}$ and : $y = \frac{2}{\pi}x$. **(06 Marks)**

b. Apply Stoke's theorem evaluate

$\int_C (x + y)dx + (2x - z)dy + (y + z)dz$ where C is the boundary of the triangle with vertices (2, 0, 0) (0, 3, 0) and (0, 0,6). **(05 Marks)**

c. Find the curve on which the functional $\int_0^1 (y')^2 + 12xy\, dx$ with y(0) = 0 and y(1) = 1 can be extremized. **(05 Marks)**

OR

10 a. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

b. Show that the geodesics on a plane are straight lines. **(05 Marks)**

c. Evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$ where $\vec{F} = 4xz\,\hat{i} + y^2\,\hat{j} + yz\,\hat{k}$ and S in the surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. **(05 Marks)**

* * * * *